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Linear Algebra MTH 221 Spring 2012, 1–2

Exam I , MTH 221 , Spring 2012

Ayman Badawi

QUESTION 1. (Write down the correct answer only in the provided space, each = 2 points, total = 42 points)

(i) Given A is a 3×4 matrix such that $A \xrightarrow{-2R_2} A_1$. Let E be an elementary matrix such that $EA_1 = A$.

Then $E = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(ii) Let $D = \begin{bmatrix} 4 & -2 \\ 1 & 2 \end{bmatrix}$. Then $D^{-1} = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} \frac{2}{10} & \frac{2}{10} \\ \frac{-1}{10} & \frac{4}{10} \end{bmatrix}$

(iii) Let $A = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$. Then $A = 0.5B + 0.5C$, where B is symmetric and C is skew-symmetric. Then

$B = \begin{bmatrix} 4 & -1 \\ -1 & 10 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 7 \\ -7 & 0 \end{bmatrix}$

(iv) Given A is 3×3 such that $A \xrightarrow{3R_2} A_1 \xrightarrow{-3R_2 + R_1 \to R_3} A_2 = \begin{bmatrix} 3 & 6 & 0 \\ 0 & 0 & 9 \\ 0 & 6 & 3 \end{bmatrix}$.

a. $det(A^T) = -54$

b. $det(A_1^{-1}) = \frac{-1}{162}$

c. $det(2A) = -432$

d. Let B an 3×3 matrix such that $BA_2 = A$. Then

B = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

e. The matrix A = → $A = \begin{bmatrix} 3 & 6 & 0 \\ 0 & 0 & 3 \\ 0 & 6 & 30 \end{bmatrix}$

(v) Let $A = \begin{bmatrix} a_1 & 7 & -5 \\ a_2 & 1 & 3 \\ a_3 & a_4 & a_5 \end{bmatrix}$. Given $det(A) = 8$. The value of x_1 when we solve the system $AX = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$ is equal to $= 13$

(1,3) entry of $A^{-1} = \frac{(7\times3)+5}{8} = \frac{13}{4}$

$x_1 = \frac{13}{4} \times 4 = 13$

$x_1 = 13$

(vi) Given A is a 3×3 matrix such that $A^{-1} = \begin{bmatrix} 2 & 4 & 1 \\ -2 & 4 & 0 \\ -4 & -8 & 0 \end{bmatrix}$

a. $(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} 2 & -2 & -4 \\ 4 & 4 & -8 \\ 1 & 0 & 0 \end{bmatrix}$

b. The (2, 3)-entry of A = $\frac{C_{3,2}}{det(A^{-1})} = \frac{-2}{32} = \frac{-1}{16}$

c. The set of solution to the system of linear equations $A^T X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

is $\{(-4, 0, 1)\}$

(vii) Let $A = \begin{bmatrix} 4 & 2 & 2 \\ -4 & b & -2 \\ 2 & 1 & c \end{bmatrix}$. Consider the system $AX = \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$. Then

a. The system will have unique solution when $b =$ $b \in \mathbb{R} \setminus \{-2\}$ and $c =$ $c \in \mathbb{R} \setminus \{1\}$

b. The system is inconsistent when $b =$ -2 and $c =$ $\in \mathbb{R}$

c. The system has infinitely many solution when $b =$ $b \in \mathbb{R} \setminus \{-2\}$ and $c =$ 1

d. If $b = -1$ and $c = 5$, then the set of solution to the system is $\{(1.5, -2, 0)\}$

(viii) Let A be a 2×2 matrix such that $(\begin{bmatrix} 2 & -4 \\ 1 & -1 \end{bmatrix} A)^{-1} = \begin{bmatrix} 0 & 4 \\ -2 & 1 \end{bmatrix}$. Then $A = \begin{bmatrix} \frac{7}{16} & \frac{4}{16} \\ \frac{3}{16} & \frac{4}{16} \end{bmatrix}$

(ix) Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}$. Given B is 3×2 matrix such that $AB = \begin{bmatrix} 4a_2 + 3a_3 & a_1 + a_2 - a_3 \\ 4a_5 + 3a_6 & a_4 + a_5 - a_6 \end{bmatrix}$. Then

$B = \begin{bmatrix} 0 & 1 \\ 4 & 1 \\ 3 & -1 \end{bmatrix}$

(x) Let F, 2×3 matrix, such that $\begin{bmatrix} 4 & 1 \\ -1 & 5 \end{bmatrix} F - 5F = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$. Then

$F = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$

(xi) Let A be a 4×4 matrix such that the 2nd column of A is identical to the 4th column of A. Let D be the 2nd column of A. Consider the system of linear equations $AX = D$. We know that the system has infinitely many solutions. Then

One particular solution is (0 , 0.5 , 0 , 0.5); second particular solution is (0 , 0.75 , 0 , 0.25);

third particular solution is (0 , 0.3 , 0 , 0.7).

(xii) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. Then

$A^{-1} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

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Exam II , MTH 221 , Spring 2012

Ayman Badawi

QUESTION 1. (19 questions, each = 2 points, Total of points = 38)

(i) let $A = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$. The eigenvalues of A are

a) -2, 1 b) 0, -1 c) 1, -1 d) None of the previous

(ii) Let A be a 2×2 matrix such that A is row-equivalent to $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$. Then the eigenvalues of A :

a) Cannot be determined b) 2, 0 c) 0.5, 0 d) None of the previous

(iii) Let A as in the previous question. Then $N(A)$ =

a) $\{(0,0)\}$ b) span$\{(0,1)\}$ c) span$\{(2,0)\}$ d) None of the previous

(iv) One of the following matrices with the given properties is diagnolizable:

a)A is 3×3, $C_A(\alpha) = (2-\alpha)^2(3-\alpha)$, and $E_2 = span\{(2,4)\}$, $E_3 = span\{(3,3)\}$

b) A is 2×2, $C_A(\alpha) = -\alpha(5-\alpha)$, $E_0 = span\{(0,1)\}$, $E_5 = span\{(3,0)\}$

c) A is 2×2, $C_A(\alpha) = \alpha^2$, $E_0 = span\{(4,1)\}$

d) None of the previous

(v) Given A is 4×4, $C_A(\alpha) = (2-\alpha)^3(5-\alpha)$. Then $det(A^T)$ =

a) 10 b) 40 c) $\frac{1}{10}$ d) $\frac{1}{40}$

(vi) Let A as above such that $A \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$. Then one of the following is true:

a) $A \begin{bmatrix} 1 \\ 0 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$

b) $A^{-1} \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1.5 \end{bmatrix}$

c) (a) and (b) correct

d) $A^{-1} \begin{bmatrix} 1 \\ 0 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$

(vii) Given A is 2×2, 0 and 1 are eigenvalues of A such that $E_0 = \{(-x_2, x_2) \mid x_2 \in R\}$ and $E_1 = \{(0, x_2) \mid x_2 \in R$. THEN (JUST WRITE DOWN THE ANSWER) $A =$

(viii) Given A is 3×3, $C_A(\alpha) = (3-\alpha)^2(a-\alpha)$. B is 3×3 and B is similar to A such that $det(B) = 54$. Then $a =$

a) 18 b) 1/18 c) 6 d) Cannot be determined

(ix) Let $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ -1 & -2 & 0 & 1 \\ 1 & 3 & 1 & 1 \end{bmatrix}$. Then $N(A) =$

a) $\{(-2x_3 - 2x_4, x_3 + x_4, x_3, x_4) \mid x_3, x_4 \in R\}$

b) $\{(2x_3, -x_3, x_3, 0) \mid x_3 \in R\}$

c) $\{(2x_3 + 2x_4, -x_3 - x_4, x_3, x_4 \mid x_3, x_4 \in R\}$ d) None of the previous.

(x) Let A as above then $N(A)$ (as span)

a) Span $\{(-2, 1, 1, 0), (-2, 1, 0, 1)\}$ b) Span$\{(-4, 2, -2, 0)\}$ c) span$\{(2, 1, 1, 0)\}$

d) Span$\{(2, -1, 1, 0), (2, -1, 0, 1)\}$ e) None of the previous

(xi) Let A as in Question ix. Column Space of A =

a) Span$\{(1, 0, 0), (0, 0, 1), (-2, 0, 1)\}$ b) Span $\{(1, -1, 1), (2, -2, 3), (0, 1, 1)\}$ c)Span$\{(1, 0, 0), (0, 0, 1), (-2, 1, 1)\}$
d) None of the previous

(xii) Let A as in Question ix. Then For each $B \in R^3$

a) The system of linear equations $AX = B^T$ has infinitely many solutions

b) The system of linear equations $AX = B^T$ has unique solution

c) The system of linear equations $AX = B^T$ may and may not be consistent (it all depends on the selected B)

d) I need more information, you keep asking nonsense questions.

(xiii) One of the following is a subspace of P_3

a) $\{(2a_1 + a_0)x^2 + a_1x + a_0 \mid a_0, a_1 \in R\}$. b) $\{a_2x^2 + x + a_0 \mid a_0, a_2 \in R\}$ c) $\{x^2 + a_1x + a_0 \mid a_0, a_1 \in R\}$

(xiv) Let D be a subspace of $R_{2\times2}$ such that $dim(D) = 2$. Then a possibility for D is

a) $D = \{\begin{bmatrix} a+b & a \\ 0 & b \end{bmatrix} \mid a, b \in R\}$ b) $D = \{\begin{bmatrix} a+1 & a \\ b & 0 \end{bmatrix} \mid a, b \in R\}$

c) $D = \{\begin{bmatrix} a+b+c & a \\ c & b \end{bmatrix} \mid a, b, c \in R\}$ d) $D = \{\begin{bmatrix} a+2b & 2a+4b \\ 0 & a+2b \end{bmatrix} \mid a, b \in R\}$

(xv) Let $D = \{(a + b + 2c, a - b, a + 2b + 3c) \mid a, b, c \in R\}$. Then $dim(D) =$

a) 1 b) 3 c)2 d) None

(xvi) Let D as in QUESTION xv. Then a basis for D

a) $\{(4, 0, 6)\}$ b) $\{(1, 1, 1), (1, 0, -1)\}$ c) $\{(1, 1, 1), (1, -1, 2)\}$ d) $\{(1, 1, 1), (1, -1, 2), (2, 0, 3)\}$

(xvii) One of the following points is in D (D is as in Question xv)

a) $(7, 1, 11)$ b) (-2, 0, 3) c) (1, 5, -1) d) (0, -2, -3)

(xviii) One of the following is a basis for P_3

a) $\{x^2, x^2 + x, x^2 + 1\}$ b) $\{x^2, x^2 + 2x + 1, 2x + 1\}$ c) $\{1, 1 + x + x^2, -1 + x + x^2\}$ d) None of the previous

(xix) Given A is a 3×3 matrix and $C_A(\alpha) = (1 - \alpha)^2(3 - \alpha)$. Given $E_1 = span\{(1, 1, 1), (1, -1, 0)\}$ and $E_3 = span\{(0, 0, -1)\}$. Let Q be a 3×3 invertible matrix such that $Q^{-1}AQ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then $Q =$

a) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$. b) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. d) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$.

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Final Exam, MTH 221: Linear Algebra, Spring 2012
Make sure you have 10 pages with 9 questions.
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QUESTION 1. (10 points) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

1. Find A^{-1}.

2. Find $(A^T)^{-1}$

3. Solve $A^T X = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

QUESTION 2. (21 points) Let A be a 3×3 matrix such that

$$A \quad \xrightarrow{-2R_1 + R_2 \to R_2} \quad A_1 \quad \xrightarrow{2R_3} \quad D = \begin{bmatrix} 2 & -1 & 2 \\ -2 & 2 & 2 \\ 4 & -2 & 6 \end{bmatrix}.$$

1. Find $det(A)$

2. Find a matrix B such that $BD = A$

3. Find $det(B)$, where B is the matrix in part (2).

4. Find $Rank(A)$

Question (2) continues:

5. Solve the system $2AX = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$

6. Find the matrix A.

7. Is there an $F \in R^3$ that will make the system of linear equations $AX = F^T$ inconsistent? If no, explain briefly. If yes, find such F

QUESTION 3. (12 points) Find a basis for the given subspace S. (They are subspaces so you do not need to check)

1. $S = span\,\{(-2,2,2),(2,0,1),(-4,4,4),(1,-1,-1)\ \}$

2. $S = \left\{ f(x) \in \mathbb{P}_3 \mid \int_0^1\ f'(x)\ dx = 0 \right\}$

3. $S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2\times 2} \mid \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$

QUESTION 4. (9 points)

1. Are $x^2, x^2 - 2, 2x^2 - 3x + 4$ independent elements of $\mathbb{P}_3$? Explain briefly your answer

2. Given, v_1, v_2, v_3 are independent points in R^n for some integer $n \geq 3$. Are $\mathbf{v}_1 - 2\mathbf{v}_2, \mathbf{v}_2 - 3\mathbf{v}_3, \mathbf{v}_3$ linearly independent points in R^n ? Explain your answer

3. Are the columns of $A = \begin{bmatrix} -1 & 0 & 2 & 1 \\ 0 & 3 & 2 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -3 & -3 \end{bmatrix}$ independent?

QUESTION 5. (12 points) Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 4 \\ 0 & 1 & 0 \end{bmatrix}$

1. For each eigenvalue of A find the corresponding eigenspace .

2. If possible diagonalize A. i.e. find an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$.

QUESTION 6. (12 points) Let $T : \mathbb{R}^3 \mapsto \mathbb{R}^3$ be a linear transformation given by $T(a, b, c) = (a - b + 3c, -a + b + c, -3a + 3b - 5c)$

1. Find a basis for the kernel of T and find its dimension.

2. Find a basis for the range of T and find its dimension.

3. Is $(1, 1, 0) \in Ker(T)$? Explain

4. Is $(4, 0, -8) \in Range(T)$? Explain

QUESTION 7. (10 points) Let $T : \mathbb{R}^2 \mapsto \mathbb{R}^3$ be a linear transformation such that $T(2,0) = (2,-4,0)$ and $(0,-1) \in Ker(T)$

1. Find $T(1,0)$ and $T(0,1)$.

2. Find the standard matrix representation of T.

3. Write Ker(T) and Range(T) as a span of basis.

QUESTION 8. (8 points) Use the Gram-Schmidt process to find an orthogonal basis for the subspace

$$W = Span\,\{(1,-2,1,1),(-1,3,-1,-1),(-1,2,-1,1)\}$$

QUESTION 9. (6 points) Let $M = \{(a, b, c) \in R^3 \mid (a, b, c).(2, -1, 2) = 0$ and $(a, b, c).(-2, 1, -1) = 0\}$. Show that M is a subspace of R^3 (i.e. Show that M satisfies the two conditions). Write M as a span of a basis of M. Note that "." means dot product.

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